Introduction to the GNU MPFR Library

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Outline

- Presentation, History
- MPFR Basics
- Output Functions
- Test of MPFR (`make check`)
- Applications
- Timings
- Conclusion

GNU MPFR in a Few Words

- GNU MPFR is an efficient multiple-precision floating-point library with well-defined semantics (copying the good ideas from the IEEE-754 standard), in particular correct rounding.
- 80 mathematical functions, in addition to utility functions (assignments, conversions. . .).
- Special data (*Not a Number*, infinities, signed zeros).
- Originally developed at LORIA, INRIA Nancy – Grand Est.
  Since the end of 2006, a joint project between the Arénaire (LIP, ENS-Lyon) and CACAO (now Caramel) INRIA project-teams.
- Written in C (ISO + optional extensions); based on GMP (mpn/mpz).
- Licence: LGPL (version 3 or later, for GNU MPFR 3).

MPFR History

1998–2000	ARC INRIA *Fiable*.
November 1998	Foundation text (Guillaume Hanrot, Jean-Michel Muller, Joris van der Hoeven, Paul Zimmermann).
Early 1999	First lines of code (G. Hanrot, P. Zimmermann).
9 June 1999	First commit into CVS (later, SVN).
June-July 1999	Sylvie Boldo (AGM, log).
2000–2002	ARC AOC (*Arithmétique des Ordinateurs Certifiée*).
February 2000	First public version.
March 2000	APP (*Agence pour la Protection des Programmes*) deposit.
June 2000	**Copyright assigned to the Free Software Foundation.**
December 2000	Vincent Lefèvre joins the MPFR team.
2001–2002	David Daney (1-year postdoc).
2003–2005	Patrick Pélissier.

MPFR History [2]

2004	GNU Fortran uses MPFR.
September 2005	MPFR 2.2.0 is released (shared library, TLS support).
October 2005	The MPFR team won the *Many Digits* Friendly Competition.
August 2007	MPFR 2.3.0 is released (shared library enabled by default).
2007–2009	Philippe Théveny.
October 2007	CEA-EDF-INRIA School *Certified Numerical Computation*.
March 2008	GCC 4.3.0 release: GCC now uses MPFR in its middle-end.
January 2009	GNU MPFR 2.4.0 is released (**now a GNU package**).
March 2009	MPFR switches to LGPL v3+ (trunk, for MPFR 3.x).
June 2010	GNU MPFR 3.0.0 is released (API clean-up).
??? 2011	GNU MPFR 3.1.0 (TLS enabled by default if supported).

Other contributions: Mathieu Dutour, Laurent Fousse, Emmanuel Jeandel, Fabrice Rouillier, Kevin Ryde, and others.

Why MPFR?

In general, exact computations on real numbers are not possible: they would be far too slow or even undecidable.

→ Different ways to provide a result:

- subset of the real numbers: fixed point, floating point (much larger range), precision (fixed/small or arbitrary);
- accuracy of the operations and functions;
- how the results are *rounded* (e.g. *correct rounding*).

Criteria:

- performance (time and memory);
- correctness (actually accuracy) and consistency;
- portability;
- reproducibility of the results (on different platforms, with different software).

Some compromise between the performance and the other criteria.
MPFR focuses on the last 3 criteria, while still being very efficient.

Example: $\sin(10^{22})$

Environment	Computed value of $\sin 10^{22}$
Exact result	$-$ **0.8522008497671888017727** ...
MPFR (53 bits)	-0.85220084976718879
Glibc 2.3.6 / x86	0.46261304076460175
Glibc 2.3.6 / x86_64	-0.85220084976718879
Mac OS X 10.4.11 / PowerPC	-0.85220084977909205
Maple 10 (Digits = 17)	-0.85220084976718880
Mathematica 5.0 (x86?)	0.462613
MuPAD 3.2.0	-0.9873536182
HP 700	0.0
HP 375, 425t (4.3 BSD)	-0.65365288 ...
Solaris/SPARC	-0.852200849 ...
IBM 3090/600S-VF AIX 370	0.0
PC: Borland TurboC 2.0	4.67734e$-$240
Sharp EL5806	-0.090748172

Note: $10^{22} = 5^{22} \times 2^{22}$, and 5^{22} fits on 53 bits.

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MPFR Program to Compute $\sin(10^{22})$

```
#include <stdio.h>   /* for mpfr_printf, before #include <mpfr.h> */
#include <assert.h>
#include <gmp.h>
#include <mpfr.h>

int main (void)
{
  mpfr_t x;  int inex;
  mpfr_init2 (x, 53);  /* x: 53-bit precision */
  inex = mpfr_set_ui (x, 10, MPFR_RNDN);     assert (inex == 0);
  inex = mpfr_pow_ui (x, x, 22, MPFR_RNDN);  assert (inex == 0);
  mpfr_sin (x, x, MPFR_RNDN);
  mpfr_printf ("sin(10^22) = %.17Rg\n", x);
  mpfr_clear (x);
  return 0;
}
```

Compile with: `gcc -Wall -O2 sin10p22.c -o sin10p22 -lmpfr -lgmp`

Representation and Computation Model

Extension of the IEEE-754 standard to the arbitrary precision:

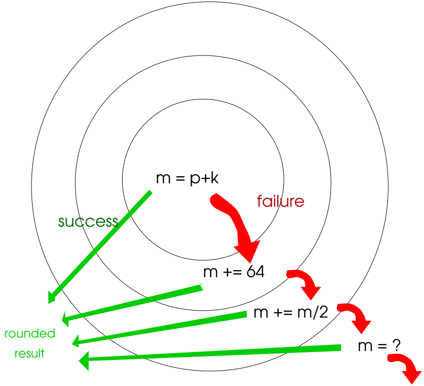
- Base 2, precision $p \geq 2$ associated with each MPFR number.
- Format of normal numbers: $\pm 0.\underbrace{1 b_2 b_3 \ldots b_p}_{p \text{ bits}} \cdot 2^e$ with $E_{\min} \leq e \leq E_{\max}$

  ($E_{\min}$ and $E_{\max}$ are chosen by the user, $1 - 2^{30}$ and $2^{30} - 1$ by default).
- No *subnormals*, but can be emulated with `mpfr_subnormalize`.
- Special MPFR data: ± 0, $\pm\infty$, NaN (only one kind, similar to sNaN).
- Correct rounding in the 4 rounding modes of IEEE 754-1985: Nearest-even, Downward, Upward, toward Zero.

  Also supports: Away from zero (new in MPFR 3.0.0).
- Correct rounding in *any* precision for *any* function. More than the accuracy, needed for reproducibility of the results and for testing arithmetics.

Caveats

- Correct rounding, variable precision and special numbers
  $\rightarrow$ noticeable overhead in very small precisions.
- Correct rounding $\rightarrow$ much slower on (mostly rare) "bad" cases (due to the *Table Maker's Dilemma*), but slightly slower in average. Ziv's strategy in MPFR:
  - first evaluate the result with slightly more precision (m) than the target (p);
  - if rounding is not possible, then $m \leftarrow m + (32 \text{ or } 64)$, and recompute;
  - for the following failures: $m \leftarrow m + \lfloor m/2 \rfloor$.



- Huge exponent range and meaningful results $\rightarrow$ functions sin, cos and tan on huge arguments are very slow and take a lot of memory.

Exceptions (Global/Per-Thread Sticky Flags)

Invalid The MPFR (floating-point) result is not defined (NaN).
Ex.: $0/0$, $\log(-17)$, but also `mpfr_set` on a NaN.

DivideByZero a.k.a. Infinitary (LIA-2). An exact infinite result is defined for a function on finite operands. **For MPFR 3.1!**
Ex.: $1/\pm 0$, $\log(\pm 0)$.

Overflow The exponent of the rounded result with unbounded exponent range would be larger than E_{max}.
Ex.: $2^{E_{\mathrm{max}}}$, and even `mpfr_set(y,x,MPFR_RNDU)` with $x = \mathrm{nextbelow}(+\infty)$ and $\mathrm{prec}(y) < \mathrm{prec}(x)$.

Underflow The exponent of the rounded result with unbounded exponent range would be smaller than E_{min}.
Ex.: If $E_{\mathrm{min}} = -17$, underflow occurs with `0.1e-17 / 2` and `0.11e-17 - 0.1e-17` (no subnormals).

Inexact The returned result is different from the exact result.

Erange Range error when the result is not a MPFR datum.
Ex.: `mpfr_get_ui` on negative value, `mpfr_cmp` on (NaN, x).

The Ternary Value

Most functions that return a MPFR number as a result (pointer passed as the first argument) also return a value of type `int`, called the *ternary value*:

- $= 0$ The value stored in the destination is exact (no rounding) or NaN.
- > 0 The value stored in the destination is greater than the exact result.
- < 0 The value stored in the destination is less than the exact result.

When not already set, the *inexact* flag is set if and only if the ternary value is nonzero.

Some Differences Between MPFR and IEEE 754

- No subnormals in MPFR, but can be emulated with `mpfr_subnormalize`.
- MPFR has only one kind of NaN (behavior is similar to signaling NaNs).
- No DivideByZero exception up to MPFR 3.0.0 (latest version).
- The Invalid exception is a bit different (see NaNs).
- Mathematical functions on special values follow the ISO C99 standard rather than IEEE 754-2008 (more recent than the MPFR specifications).
- Memory representation is different, but the mapping of a bit string (specified by IEEE 754) into memory is implementation-defined anyway.
- Some operations are not implemented.
- And other minor differences...

Memory Handling

- Type `mpfr_t`: `typedef __mpfr_struct mpfr_t[1];`
  - when a `mpfr_t` variable is declared, the structure is automatically allocated (the variable must still be initialized with `mpfr_init2` for the significand);
  - in a function, the pointer itself is passed, so that in `mpfr_add(a,b,c,rnd)`, the object `*a` is modified;
  - associated pointer: `typedef __mpfr_struct *mpfr_ptr;`
- MPFR numbers with more precision can be created internally.
  Warning! Possible crash in extreme cases (like in most software).
- Some MPFR functions may create caches, e.g. when computing constants such as π. Caches can be freed with `mpfr_free_cache`.
- MPFR internal data (exception flags, exponent range, caches...) are either global or per-thread (if MPFR has been built with TLS support).

Logging

When MPFR has been built with `-enable-logging` (on supported platforms), environment variables can be defined for logging:

MPFR_LOG_FILE	Name of the log file (default: `mpfr.log`).
MPFR_LOG_PREC	Number of digits of the output (default: 6).
MPFR_LOG_LEVEL	Max recursive level (default: 7).
MPFR_LOG_INPUT	Log the function input.
MPFR_LOG_OUTPUT	Log the function output.
MPFR_LOG_TIME	Log the time spent inside the function.
MPFR_LOG_INTERNAL	Log some particular variables if any.
MPFR_LOG_MSG	Log the messages if any.
MPFR_LOG_ZIV	Log what the Ziv loops do.
MPFR_LOG_STAT	Log how many times a Ziv loop failed.
MPFR_LOG_ALL	Log everything.

Output Functions

	Simple output	Formatted output
To file	`mpfr_out_str`	`mpfr_fprintf`, `mpfr_printf`
To string	`mpfr_get_str`	`mpfr_sprintf`
MPFR version	old	2.4.0
Locale-sensitive	yes (2.2.0)	yes
Base	2 to 36 (2.x) 2 to 62 (3.x)	2, 10, 16
Read-back exactly	yes (prec $= 0$)	yes[1] (empty precision field)
Efficiency	-	very slow in base 10 (fixed for MPFR 3.1.0)

[1]Except for the conversion specifier `g` (or `G`) — documentation of MPFR 2.4.1 is incorrect.

Simple Output (`mpfr_out_str`, `mpfr_get_str`)

```
size_t mpfr_out_str (FILE *stream, int base, size_t n,
                     mpfr_t op, mp_rnd_t rnd)
```

Base b: from 2 to 62 (from 2 to 36 before MPFR 3.0.0).

Precision n: number of digits or 0. If $n = 0$:

- The number of digits m is chosen large enough so that re-reading the printed value with the same precision, assuming both output and input use rounding to nearest, will recover the original value of op.
- More precisely, if p is the precision of op, then $m = \lceil p . \log(2) / \log(b) \rceil$, and $m = \lceil (p - 1) . \log(2) / \log(b) \rceil$ when b is a power of 2 (it has been check that these formulas are computed exactly for practical values of p).

Output to string: `mpfr_get_str` (on which `mpfr_out_str` is based).

Formatted Output Functions (printf-like)

Conversion specification:

```
% [flags] [width] [.[precision]] [type] [rounding] conv
```

Examples (32-bit $x \approx 10000/81 \approx 123.45679012$):

```
  mpfr_printf ("%Rf %.6RDe %.6RUe\n", x, x, x);
> 123.45679012 1.234567e+02 1.234568e+02
  mpfr_printf ("%11.1R*A\n", MPFR_RNDD, x);
>    0X7.BP+4
  mpfr_printf ("%.*Rb\n", 6, x);
> 1.111011p+6
  mpfr_printf ("%.9Rg %#.9Rg\n", x, x);
> 123.45679 123.456790
  mpfr_printf ("%#.*R*g %#.9g\n", 8, MPFR_RNDU, x, 10000./81.);
> 123.45680 123.456790
```

Test of MPFR (make check)

In the GCC development mailing-list, on 2007-12-29:

http://gcc.gnu.org/ml/gcc/2007-12/msg00707.html

```
> On 29 December 2007 20:07, Dennis Clarke wrote:
>
>>
>> Do you have a testsuite ?  Some battary of tests that can be thrown at the
>> code to determine correct responses to various calculations, error
>> conditions, underflows and rounding errors etc etc ?
>
> There's a "make check" target in the tarball.  I don't know how thorough
> it is.

That is what scares me.

Dennis
```

Test of MPFR (`make check`) [2]

Exhaustive testing is not possible.
→ Particular and generic tests (random or not).

- Complete branch coverage (or almost), but not sufficient.
- Function-specific or algorithm-specific values and other difficulties (e.g., based on bugs that have been found).
  1. Bug found in some function.
  2. Corresponding particular test added.
  3. Analysis:
     - Reason of the bug?
     - Can a similar bug be found somewhere else in the MPFR code (current or future)?
  4. Corresponding generic test(s) added.
- Tests with various gcc options, with valgrind.

In addition to `make check`, potential bugs detected by mpfrlint.

What Is Tested

- Special data in input or output: NaN, infinities, ± 0.
- Inputs that yield exceptions, exact cases, or midpoint cases in rounding-to-nearest.
- Discontinuity points.
- Bit patterns: for some functions (arithmetic operations, integer power), random inputs with long sequence of 0's and/or 1's.
- Thresholds: *hard-to-round cases*, underflow/overflow thresholds (currently for a few functions only).
- Extreme cases: tiny or huge input values.
- Reuse of variables (`reuse.c`), e.g. in `mpfr_exp(x,x,rnd)`.
- The influence of previous data: exception flags, sign of the output variable.
- Weird exponent range, e.g. $[17, 59]$.

The Generic Tests (`tgeneric.c`)

Basic Principle

A function is first evaluated on some input x in some target precision $p + k$, and if one can deduce the result in precision p (i.e., the TMD does not occur), then one evaluates f on the same input x in the target precision p, and compare the results.

- The precision p and the inputs are chosen randomly (in some ranges). Special values (tiny and huge inputs) can be tested too.
- Functions with 2 inputs (possibly integer) are supported.
- The exceptions are supported (with a consistency test of flags and values).
- The ternary value is checked.
- The evaluations can be performed in different flag contexts (to check the sensitivity to the flags).
- **New:** An evaluation can be redone in an extremely reduced exponent range.
- In the second evaluation, the precision of the inputs can be increased.
- The exponent range is checked at the end (bug if not restored).

Testing Bad Cases for Correct Rounding (TMD)

- Small-precision worst cases found by exhaustive search (in practice, in double precision), by using function `data_check` of `tests.c`. These worst cases are currently *not* in the repository. Each hard-to-round case is tested
  - in rounding-to-nearest, in target precision $p - 1$,
  - in all the directed rounding modes in target precision p,

  where p is the minimal precision of the corresponding *breakpoint*.

- Random hard-to-round cases (when the inverse function is implemented), using the fact that the input can have more precision than the output (function `bad_cases` of `tests.c`):
  1. A precision p_y and a MPFR number y of precision p_y are chosen randomly.
  2. One computes $x = f^{-1}(y)$ in a precision $p_x = p_y + k$.
     $\rightarrow$ In general, x is a bad case for f in precision p_y for directed rounding modes (and rounding-to-nearest for some smaller precision).
  3. One tests x in all the rounding modes (see above).

  TODO: use Newton's iteration for the other functions?

Application 1: Test of Sum Rounded to Odd

Algorithm OddRoundedAdd

function $z = \text{OddRoundedAdd}(x, y)$

$$
\begin{aligned}
d &= \text{RD}(x + y); \\
u &= \text{RU}(x + y); \\
e' &= \text{RN}(d + u); \\
e &= e' \times 0.5; && \{ \text{ exact } \} \\
z &= (u - e) + d; && \{ \text{ exact } \}
\end{aligned}
$$

This algorithm returns the sum $z = x + y$ rounded-to-odd:

- $\text{RO}(z) = z$ if z is a machine number;
- otherwise $\text{RO}(z)$ is the value among $\text{RD}(z)$ and $\text{RU}(z)$ whose least significant bit is a one.

The corresponding MPFR instructions:

```
mpfr_add (d, x, y, MPFR_RNDD);
mpfr_add (u, x, y, MPFR_RNDU);
mpfr_add (e, d, u, MPFR_RNDN);
mpfr_div_2ui (e, e, 1, MPFR_RNDN);
mpfr_sub (z, u, e, MPFR_RNDN);
mpfr_add (z, z, d, MPFR_RNDN);
```

Application 1: Test of Sum Rounded to Odd [2]

```
#include <stdio.h>
#include <stdlib.h>
#include <gmp.h>
#include <mpfr.h>

#define LIST x, y, d, u, e, z

int main (int argc, char **argv)
{
  mpfr_t LIST;
  mp_prec_t prec;
  int pprec;       /* will be prec - 1 for mpfr_printf */

  prec = atoi (argv[1]);
  pprec = prec - 1;

  mpfr_inits2 (prec, LIST, (mpfr_ptr) 0);
```

Application 1: Test of Sum Rounded to Odd [3]

```
if (mpfr_set_str (x, argv[2], 0, MPFR_RNDN))
  {
    fprintf (stderr, "rndo-add: bad x value\n");
    exit (1);
  }
mpfr_printf ("x = %.*Rb\n", pprec, x);

if (mpfr_set_str (y, argv[3], 0, MPFR_RNDN))
  {
    fprintf (stderr, "rndo-add: bad y value\n");
    exit (1);
  }
mpfr_printf ("y = %.*Rb\n", pprec, y);
```

Application 1: Test of Sum Rounded to Odd [4]

```
  mpfr_add (d, x, y, MPFR_RNDD);
  mpfr_printf ("d = %.*Rb\n", pprec, d);

  mpfr_add (u, x, y, MPFR_RNDU);
  mpfr_printf ("u = %.*Rb\n", pprec, u);

  mpfr_add (e, d, u, MPFR_RNDN);
  mpfr_div_2ui (e, e, 1, MPFR_RNDN);
  mpfr_printf ("e = %.*Rb\n", pprec, e);

  mpfr_sub (z, u, e, MPFR_RNDN);
  mpfr_add (z, z, d, MPFR_RNDN);
  mpfr_printf ("z = %.*Rb\n", pprec, z);

  mpfr_clears (LIST, (mpfr_ptr) 0);
  return 0;
}
```

Application 2: Test of the Double Rounding Effect

Arguments: $d_{\max}$, target precision n, extended precision p (by default, $p = n$).

Return all the couples of positive machine numbers (x, y) such that $1/2 \leq y < 1$, $0 \leq E_x - E_y \leq d_{\max}$, $x - y$ is exactly representable in precision n and the results of $\lfloor \circ_n(\circ_p(x/y)) \rfloor$ in the rounding modes toward 0 and to nearest are different.

```
#include <stdio.h>
#include <stdlib.h>
#include <mpfr.h>

#define PRECN x, y, z  /* in precision n, t in precision p */

static unsigned long
eval (mpfr_t x, mpfr_t y, mpfr_t z, mpfr_t t, mpfr_rnd_t rnd)
{
  mpfr_div (t, x, y, rnd);  /* the division x/y in precision p */
  mpfr_set (z, t, rnd);     /* the rounding to the precision n */
  mpfr_rint_floor (z, z, rnd);  /* rnd shouldn't matter */
  return mpfr_get_ui (z, rnd);  /* rnd shouldn't matter */
}
```

Application 2: Test of the Double Rounding Effect [2]

```
int main (int argc, char *argv[])
{
  int dmax, n, p;
  mpfr_t PRECN, t;

  if (argc != 3 && argc != 4)
    { fprintf (stderr, "Usage: divworst <dmax> <n> [ <p> ]\n");
      exit (EXIT_FAILURE); }

  dmax = atoi (argv[1]);
  n = atoi (argv[2]);
  p = argc == 3 ? n : atoi (argv[3]);
  if (p < n)
    { fprintf (stderr, "p must be greater or equal to n\n");
      exit (EXIT_FAILURE); }

  mpfr_inits2 (n, PRECN, (mpfr_ptr) 0);
  mpfr_init2 (t, p);
```

Application 2: Test of the Double Rounding Effect [3]

```
  for (mpfr_set_ui_2exp (x, 1, -1, MPFR_RNDN);
       mpfr_get_exp (x) <= dmax; mpfr_nextabove (x))
    for (mpfr_set_ui_2exp (y, 1, -1, MPFR_RNDN);
         mpfr_get_exp (y) == 0; mpfr_nextabove (y))
      {
        unsigned long rz, rn;

        if (mpfr_sub (z, x, y, MPFR_RNDZ) != 0)
          continue;  /* x - y not representable in precision n */
        rz = eval (x, y, z, t, MPFR_RNDZ);
        rn = eval (x, y, z, t, MPFR_RNDN);
        if (rz != rn)
          mpfr_printf ("x = %.*Rb ; y = %.*Rb ; Z: %lu ; N: %lu\n",
                       n - 1, x, n - 1, y, rz, rn);
      }

  mpfr_clears (PRECN, t, (mpfr_ptr) 0);
  return 0;
}
```

Application 3: Continuity Test

Compute $f(1/2)$ in some given (global) precision for
$f(x) = (g(x) + 1) - g(x)$ and $g(x) = \tan(\pi x)$.

```
#include <stdio.h>
#include <stdlib.h>
#include <mpfr.h>

int main (int argc, char *argv[])
{
  mp_prec_t prec;
  mpfr_t f, g;

  if (argc != 2)
    {
      fprintf (stderr, "Usage: continuity2 <prec>\n");
      exit (EXIT_FAILURE);
    }
```

Application 3: Continuity Test [2]

```
  prec = atoi (argv[1]);
  mpfr_inits2 (prec, f, g, (mpfr_ptr) 0);

  mpfr_const_pi (g, MPFR_RNDD);
  mpfr_div_2ui (g, g, 1, MPFR_RNDD);
  mpfr_tan (g, g, MPFR_RNDN);

  mpfr_add_ui (f, g, 1, MPFR_RNDN);
  mpfr_sub (f, f, g, MPFR_RNDN);
  mpfr_printf ("g(1/2) = %-17Rg  f(1/2) = %Rg\n", g, f);

  mpfr_clears (f, g, (mpfr_ptr) 0);
  return 0;
}
```

Application 3: Continuity Test [3]

```
Precision  2    g(1/2) = 16              f(1/2) = 0
Precision  3    g(1/2) = 14              f(1/2) = 2
Precision  4    g(1/2) = 14              f(1/2) = 1
Precision  5    g(1/2) = 120             f(1/2) = 0
Precision  6    g(1/2) = 120             f(1/2) = 0
Precision  7    g(1/2) = 121             f(1/2) = 1
Precision  8    g(1/2) = 2064            f(1/2) = 0
Precision  9    g(1/2) = 2064            f(1/2) = 0
Precision 10    g(1/2) = 2068            f(1/2) = 0
Precision 11    g(1/2) = 2066            f(1/2) = 2
Precision 12    g(1/2) = 2067            f(1/2) = 1
Precision 13    g(1/2) = 4172            f(1/2) = 1
Precision 14    g(1/2) = 8502            f(1/2) = 1
Precision 15    g(1/2) = 17674           f(1/2) = 1
Precision 16    g(1/2) = 38368           f(1/2) = 1
Precision 17    g(1/2) = 92555           f(1/2) = 1
Precision 18    g(1/2) = 314966          f(1/2) = 2
```

Application 3: Continuity Test [4]

```
Precision 19    g(1/2) = 314967          f(1/2) = 1
Precision 20    g(1/2) = 788898          f(1/2) = 1
Precision 21    g(1/2) = 3.18556e+06     f(1/2) = 0
Precision 22    g(1/2) = 3.18556e+06     f(1/2) = 1
Precision 23    g(1/2) = 1.32454e+07     f(1/2) = 2
Precision 24    g(1/2) = 1.32454e+07     f(1/2) = 1
Precision 25    g(1/2) = 6.29198e+07     f(1/2) = 2
Precision 26    g(1/2) = 6.29198e+07     f(1/2) = 1
Precision 27    g(1/2) = 1.00797e+09     f(1/2) = 0
Precision 28    g(1/2) = 1.00797e+09     f(1/2) = 0
Precision 29    g(1/2) = 1.00797e+09     f(1/2) = 2
Precision 30    g(1/2) = 1.00797e+09     f(1/2) = 1
Precision 31    g(1/2) = 1.64552e+10     f(1/2) = 0
Precision 32    g(1/2) = 1.64552e+10     f(1/2) = 0
Precision 33    g(1/2) = 1.64552e+10     f(1/2) = 0
Precision 34    g(1/2) = 1.64552e+10     f(1/2) = 1
Precision 35    g(1/2) = 3.90115e+11     f(1/2) = 0
```

Timings

Maple	Mathematica	Sage	GMP MPF	MPFR	PARI	NTL	CLN
commercial	commercial	GPL	LGPL	LGPL	GPL	GPL	GPL
12.00	6.0.1	4.5.2	5.0.1	3.0.0	2.4.2.alpha	5.5.2	1.3.1
interactive	interactive	interactive	library	library	library	library	library

100 digits	Maple	Mathematica	Sage	MPF	MPFR	Pari	NTL	CLN
mult	0.0020	0.0006	0.00053	**0.00011**	0.00012	0.00013	0.000367	0.000174
div	0.0029	0.0017	0.00076	**0.00031**	0.00032	0.00034	0.00070	0.000486
sqrt	0.032	0.0018	0.00132	0.00055	**0.00049**	0.00050	0.00442	0.00068
exp	0.070	0.019	0.0103	na	**0.0083**	0.0112	0.069	0.0194
log	0.100	0.028	0.0173	na	**0.0102**	0.0120	0.386	0.0279
sin	0.131	0.017	0.0112	na	**0.0070**	0.0105	0.074	0.0250
cos	0.119	0.018	0.0078	na	**0.0052**	0.0091	0.082	0.0212
acos	0.450	0.053	0.058	na	0.044	**0.028**	na	0.032
atan	0.280	0.048	0.051	na	0.037	**0.026**	na	0.028

Timings [2]

1000 digits	Maple	Mathematica	Sage	MPF	MPFR	Pari	NTL	CLN
mult	0.0200	0.007	0.0039	0.0036	**0.0028**	0.0035	0.0137	0.0036
div	0.0200	0.015	0.0071	**0.0040**	0.0058	0.0059	0.0201	0.0079
sqrt	0.160	0.011	0.0064	0.0049	**0.0047**	**0.0047**	0.187	0.0063
exp	0.90	0.63	0.208	na	**0.182**	0.364	5.96	0.330
log	0.300	0.67	0.195	na	**0.161**	0.204	48.1	0.400
sin	1.89	0.41	0.210	na	**0.192**	0.310	6.78	0.288
cos	1.91	0.40	0.190	na	**0.181**	0.298	6.98	0.269
acos	2.50	0.82	0.81	na	**0.38**	0.75	na	0.48
atan	2.10	0.80	0.70	na	**0.36**	0.74	na	0.45

10000 digits	Maple	Mathematica	Sage	MPF	MPFR	Pari	NTL	CLN
mult	0.80	0.28	0.11	0.107	**0.095**	0.109	0.508	0.107
div	0.80	0.56	0.28	**0.198**	0.261	0.264	1.662	0.454
sqrt	3.70	0.36	0.224	0.179	**0.176**	**0.176**	20.48	0.295
exp	50.0	17.6	9.6	na	**9.1**	12.5	1560	13.4
log	20.0	15.9	7.6	na	**7.2**	8.3	16080	16.7
sin	93.0	44.4	17.3	na	**15.6**	21.7	1650	17.8
cos	92.0	44.4	17.1	na	**15.7**	21.0	7710	16.7
acos	87.0	91.2	29.4	na	**16.8**	31.7	na	28.6
atan	82.0	87.2	26.4	na	**15.3**	30.2	na	27.0

Support

- MPFR manual in info, HTML and PDF formats (if installed).
- MPFR web site: `http://www.mpfr.org/` (manual, FAQ, patches…).
- MPFR project page: `https://gforge.inria.fr/projects/mpfr/` (with Subversion repository).
- Mailing-list `mpfr@loria.fr` with
  - official archives: `http://websympa.loria.fr/wwsympa/arc/mpfr`;
  - Gmane mirror: `http://dir.gmane.org/gmane.comp.lib.mpfr.general`.

  45 messages per month in average.

How To Contribute to GNU MPFR

- Improve the documentation.
- Find, report and fix bugs.
- Improve the code coverage and/or contribute new test cases.
- Measure and improve the efficiency of the code.
- Contribute a new mathematical function.
  - Assign (you or your employer) the copyright of your code to the FSF.
  - Mathematical definition, specification (including the special data).
  - Choose one or several algorithms (with error analysis).
  - Implementation: conform to ISO C89, C99, and GNU Coding Standards.
  - Write a test program in `tests` (see slides on the tests).
  - Write the documentation (`mpfr.texi`), including the special cases.
  - Test the efficiency of your implementation (optional).
  - Send your contribution as a patch (obtained with `svn diff`).

More information: `http://www.mpfr.org/contrib.html`

The Future (MPFR 3.1)

- TLS support is now detected automatically. If TLS is supported, MPFR is built as thread safe by default.
- Much faster formatted output (`mpfr_printf`, etc.) with %Rg and similar.
- New division-by-zero exception (flag) and associated functions.
- The `mpfr.h` header can be included several times.
- Logging: `mpfr_fprintf` is now used instead of `fprintf` with the GNU libc `register_printf_function` extension.
- Static assertions?
- Other small changes.

MPFR 3.1.0 is planned for October 2011.

Other Projects Based on MPFR

GNU MPFR does not track the errors, though this is partly done internally to implement correct rounding. Other software can be used for this purpose:

- Norbert Müller's C++ package iRRAM (no longer maintained) implements an exact real arithmetic (with limitations).
- Alternatively, interval arithmetic can be used: MPFI. An exact value x is represented by a pair $(\underline{x}, \overline{x})$ such that $x \in [\underline{x}, \overline{x}]$ (inf-sup representation).
- MIRAMAR project: mid-rad interval arithmetic, where an exact value x (real or complex) is represented by a pair (m, r), m being an approximation to x (in arbitrary precision) and r an error bound in small precision.

  INRIA is recruiting an engineer (graduated in 2010 or 2011) for this project. Application deadline: September 30, 2011.

GNU MPFR does not support complex numbers.

This is the goal of GNU MPC...